B.C.A .

PART - II

SUBJECT
STATISTICS AND LINEAR PROGRAMME TECHNIQUE

PAPER - XV

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THE OBJECTIVE

After studing this lesson, you should be able to :

* Able to formulate the problems management in different area of business, government, industry, hospitals, libraries, etc and to form LPP model.

* LPP model provides a logical and synthetic approach to the problem

* Applying LP technique to identify the problem

* Express objective function and resource consraints in LP model in terms of decision constraint and parameters in real life

* Helps in finding solutions for resarch and improvement in a system.

* LPP model clearly shows the limitations and scope of an activity

INTRODUCTION

Linear Programming is one of the most versatile, powerful and useful techniques for making managerial decisions. Linear programming teachnique may be used for solving broad range of problems arising in business, government, industry, hospitals, libraries, etc and optimize the problems.

Whenever we want to allocate the available limited resources for various competing activities for achieving our desired objective, the technique that helps us is LINEAR PROGRAMMIG. As a decision making tool, it has demonstrated its value in various field such as production, finance, marketing, research and development and personnel management. Determination of optimal product mix (A combination of products, which gives maximum profit), transportation schedules, Assignment problem and many more.

In 1947, George Donting developed an efficient method called ‘Simplex algorithm’ for solving the Linear Programme.

The method is primarily used for solving military logistic problems and optimization problems in industries such as banking, education, transportation, efficient manufacturing, etc. Mainly the Linear Programming modeling technique to applied for solving a real life decision problems. While making a decision are must be aware of all these properties and assumptions. For a firm, optimization essential means maximization of profit or minimization of cost which eventually leads to maximization of profit.

The real life problems of maximization and minimization that a firm has to face are very complex because in actual business operations a firm has to deal with a large number of variable with several constraints. In their attempt to arrive at an optimized solution to the problems of resources, a good knowledge of Linear Programming modeling technique is required.

It is important to first understand the meaning of words linear and programming.

* The word linear refers to Linear relationships among variables in a model. Thus, a given change in one variable will always result in a proportional change in another variable.

* The word programming referes to mathematical modeling and of a problems that involves the economic allocation of limited resources by choosing a particular course of action or strategy among various alternative strategies in order to achieve the desired objective.

In this unit, let us discuss about various types of linear proramming models, advantages and its limitations. And further discuss the basic, non-basic variables, feasible and basic feasible solution.

MATHEMATICAL MODEL FORMULATION

It is important to organise a problem which can be handled by the method of linear programming and to formulate its mathematical model.

The steps for mathematical formulation of a Linear Programming Problem (LPP) are :

(i) The objecctive functions, the set of constraints and the non-negative restrictions together form a linear programming problem (LPP).

The general structure of an LPP model consists of these three basic components.

(ii) Decision variables (Activity) : We need to evaluate various alternatives for arriving at the optimal value of objective function. If ther are no alternatives to select from, we would no need an LP.

The decision variables usually are denoted b x_1, x_2, x_3, ..., x_n.

In an LP model all decision variable are continuous, controllable and non-negative that is $x_1 > 0$, $x_2 > 0$, $x_3 > 0$, ..., $x_n > 0$.

The objective function : The objective function of each LP problem is expressed in terms of decision variables by optimizing the criteria of optimality (also called measure of performance) such as profit, loss, revenue, distance, etc.

General form :

Optimize (Maximize or Minimize)

$$Z = c_1x_1 + c_2x_2 + ... + c_nx_n$$

Where, Z is the measure of performance variable, Which is a function of $x_1x_2.....x_n$ quantinty $c_1,c_2,.....,c_n$ are parameters that represent the contribution of a unit of the respective variables, x_1, x_2, ..., x_n to the measure of performance i.e. Z.

(iii) To formulate the other conditions of the problem such as resource limitation, market constraints, interrelations between variables etc. as Linear inequation or equation in terms of the decision variables.

(iv) To add the non-negative constraints from the considerations so that the negative value of the decision variables do not have any valid physical interpretation.

EXAMPLE 1.1

A firm manufactures 3 products A, B & C. The profits are Rs. 2, Rs. 3 and Rs 4 respectively.

The firm has 2 machines and below is the required processing time in hours on each machine for each product.

Machine	Procucts		
	A	B	C
M1	40	30	20
M2	25	25	40

Machine M1 and M2 have 100 and 150 machine hours respectively. Set up a mathematical model.

Soln : Let x = number of units of product A

y = ,, ,, ,, ,, B

z = ,, ,, ,, ,, C

(x, y, z) are decision variables)

Machine	Products			Machine hours available
	A (x)	B (y)	C (z)	
M1	40	30	20	100
M2	25	25	40	150
Profit	3	2	4	

Objective function

$Z = 3x + 2y + 4z$ ------------- (1)

(total profit earned by manufaturer)

total number of machine hours to produce x units of A, y units of B and Z units of C on Machine M_1 is

$40x + 30y + 20z$

Since machine hours available on machine M_1 cannot exceed 100

$\therefore$ $40x + 30y + 20z \leq 100$ ----------- (2)

Similarly, total number of machine hours available on machine M_2 is

$25x + 25y + 40z \leq 150$ ------- (3)

From (1), (2), (3) the mathematical model of the problem Is :

Find x, y, z which minimize

$Z = 3x + 2y + 4z$

Constraints (subject to condition) :

$40x + 30y + 20z \leq 100$

$25x + 25y + 40z \leq 150$

also, $x \geq 0, y \geq 0, z \geq 0;$

Since the fim cannot produce negative quantities (non-negative restriction)

Example 1.2

Make the mathematical form for the following problem.

A manufacturer of furniture makes two products : beds and sofas

Their processing is done on two machines A & B .

The sofa requires 5 hours on machine A and no time on machine B. The bed requires 2 hours on machine A and 6 hours on machine B. There are 20 hours of time per day available on machine A and 15 hours on machine B.

Profit gained to the manufacturer from a bed and a sofa is Rs. 20 and Rs. 30 respectively. What should be the daily production of each of the two products so that the profit may be maximum.

Solution :

Let x and y be the decision variables

where, x = number of beds produced

y = number of sofas produced.

Machine	bed (x)	sofa (y)	number of hours available
A	2	5	20
B	6	0	15
Profit	20	30	

thus, the mathematical model will be find x and y which maximize

objective function :

$Z = 20x + 30y$

constraints :

$2x + 5y \leq 20$

$6x \leq 15$

$x \geq 0$, $y \geq 0$; since firm annot produce negative quantities.

(non-negative restriction)

Example 1.3 :

A manufacturer produces two types of models M_1 and M_2. Each model of the type M_1 requires 4 hours of grinding and 2 hours of polishing; where as each model of the type M_2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works 40 hours a week and each polisher works for 60 hours a week. Profit on M_1 model is Rs. 3.00 and on model M_2 is Rs. 4.00. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models, so that he may make the maximum profit in a week ?

Solution :

Let x_1 and x_2 be the decision variables where they denote number of units of M_1 and M_2 models, respectively.

objective function (maximize the profit)

Max $Z = 3x_1 + 4x_2$

Constraints :

There are two constraints : One for grinding and the other for polishing.

No. of hours available on each grinder for one week is 40. There are 2 grinders. Hence the manufacturer doesnot have more than $2 \times 40 = 80$ hours of grinding. M_1 requires 4 hours of grinding.

the grinding constraint is given by :

$4x_1 + 2x_2 \leq 80$

Since there are 3 polishers, the available time for polishing in a week is given by $3 \times 60 = 180$ hours of polishing. M_1 requires 2 hours of polishing and M_2 requires 5 hours of polishing. Hence, we have $2x_1 + 5x_2 \leq 180$

finaly we have, max $Z = 3x_1 + 4x_2$

subject to, $4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$

$x_1, x_2 \geq 0$ (non-negative restriction)

Example 1.4 :

A person requires 10, 12 and 12 units of chemicals A, B, C respectively, for his garden A liquid product contains 5, 2 and 1 units of A, B and C respecively, per jar. A dry product contains 1, 2 and 4 units of A, B, C per carton. If the liquid product sells for Rs. 3 per jar and the dry product sells for Rs. 2 per carton, how many of each should be purchased, in order to minimize the cost and meet the requirement.

Solution :

Let x_1 and x_2 be the decision variables

Where, x_1 = number of unis of liquid

x_2 = number of units of dry products

Objective function (minimize the cost) :

Min $Z = 3x_1 + 2x_2$

constraints : there are 3 constraints for the three chemicals.

$5x_1 + x_2 \geq 10$

$2x_1 + 2x_2 \geq 12$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$ (non-negative restriction)

Example 1.5 :

A paper will produces two grades of paper namely x and y. Owing to raw material restrictions. It cannot produce more than 400 tons of grade x and 300 tons of grade y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 to produce a ton of products x and y respectively with corresponding profits of Rs. 200 and Rs. 500 per ton. Formulate the above as a LPP to maximize profit and find the optimum product min.

Solution :

Let x_1 and x_2 be be the decision variables

Where, x_1 = number of units of two grades of paper of x

x_2 = number of units of two grades of paper of y

objective function (maximize the profit) :

Max $Z = 200x_1 + 500x_2$

Constraints : two constrains are there, one for raw material and other for production hours

$x_1 \leq 400$

$x_2 \leq 300$

$0.2x_1 + 0.4x_2 \leq 160$

$x_1, x_2 \geq 0$ (non-negative restriction)

Example 1.6 :

A company manufactures two products A and B takes twice as long produce as one unit of A and if the company was to produce only A, it would have time to produce 2000 units per day. The availability of the raw material is sufficient to produce 1500 units per day of both A and B combined. Product B requiring a special ingredient, only 600 units can be made per day. If A fetches a profit of Rs. 2 per unit and B a profit of Rs. 4 per unit, find he optimum product min.

Solution :

Let x_1 and x_2 be decision variables

Where, x_1 = number of units of the products A

x_2 = number of units of the products B

objective function (Maximize the profit)

Max $Z = 2x_1 + 4x_2$

Constraints :

$x_1 + 2x_2 \leq 2000$

$x_1 + x_2 \leq 1500$

$x_2 \leq 600$

$x_1, x_2 \geq 0$ (non-negative restriction)

Example 1.7 :

A firm manufactures 3 products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has 2 machines and given below is the required processing time in minutes for each machine on each product.

Machine	Products A	B	C
M1	4	3	5
M2	3	2	4

Machines M1 and M2 have 2000 and 2500 machine minutes respectively. The firm must manufacture 100 a's, 200 B's and 50 C's but no more than 150 A's. Set up an LP problem to maximize the profit.

Solution :

Let x_1, x_2, x_3 be the number of units of products A, B, C respectively.

(x_1, x_2, x_3 are decision variables)

objective function (Maximize the profit) :

$Z = 3x_1 + 2x_2 + 4x_3$

Constraints :

$4x_1 + 3x_2 + 5x_3 \leq 2000$

$3x_1 + 2x_2 + 4x_3 \leq 2500$

here since the firm manufactures 100 A's, 200 B's and 50 C/s but not more than 150 A's the further restriction becomes

$100 \leq x_1 \leq 150$

$200 \leq x_2 \geq 0$

$50 \leq x_3 \geq 0$

Hence the complete LPP will be :

find the value of x_1, x_2, x_3 so as

to maximize

$Z = 3x_1 + 2x_2 + 4x_3$

subject o the constraints $x_1, x_2 \geq 0$

$4x_1 + 3x_2 + 5x_3 \leq 2000$

$3x_1 + 2x_2 + 4x_3 \leq 2500$

$100 \leq x_1 \leq 150,\ 200 \leq x_2 \geq 0,\ 50 \leq x_3 \geq 0$